Homotrinuclear Spin Cluster with Orbital Degeneracy in a Magnetic Field: Algebraic Dynamic Studies of the Geometric Phase

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Based on the homotrinuclear spin cluster having SU(2)⊗SU(2) symmetry with twofold orbital degeneracy $\tau = 1/2$) and the SU(2) algebraic structures of both $\hat{s}$ and $\hat{\tau}$ subspaces in the external magnetic field, we calculate exactly the non-adiabatic energy levels and the cyclic and non-cyclic non-adiabatic geometric phase of the homotrinuclear spin cluster by making use of the method of algebraic dynamics. The solution will show that the Berry phase is much influenced by the parameters $N = \gamma_s/\gamma_\tau$ (γ_s and γ_τ are the magnetic momentums of $\hat{s}$ and $\hat{\tau}$ subspaces, respectively) in addition to ω/Ω in a rotating magnetic field. The change of the Berry phase in the basis state of the system is demonstrated from the changing diagram.

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1. Introduction

In recent years, a lot of attention has been paid to the cluster system [1 – 5], because the cluster system has many complicated properties. Most of these investigations only considered a pure spin space. In contrast to the pure spin system, the spin cluster system which has orbital degeneracy can be described as a direct product of $\hat{s}$ subspace and $\hat{\tau}$ subspace and has SU(2)⊗SU(2) symmetry [6, 7]. Orbital degeneracy appears in the electron configuration of many transitional metal oxides, such as vanadium oxides [8 – 10] and manganese oxides [11]. In these systems, the orbital occupancy has an important influence on the magnitude and sign of spin-spin couplings. By their research on vanadium sesquioxide V_2O_3, Castellani et al. [8] have shown a Hamiltonian that describes spin $s = 1/2$ systems with twofold degenerate orbits (isospin $\tau = 1/2$). This is believed to be relevant to the unusual properties of transitional metal oxides and the Mott insulating phase. The isotropic limit of a spin system with orbital degeneracy which has global SU(4) symmetry has been studied by Li et al. [7]. In this model, the Hamiltonian is rotationally symmetric in both $\hat{s}$ and $\hat{\tau}$ subspaces. Furthermore, a new possibility for the spin liquid ground state in higher dimensions was provided by this model. In [7] it has also been shown that the two neighbouring sites will have anti-ferromagnetic spin-spin correlations in the SU(4) Néel ordered state.

Dynamic algebraic can be a powerful method to study the system by means of the dynamical group [12, 13]. The algebraic dynamic method is a quantum mechanics theory, which deals with the dynamic evolution based on the algebraic structure of the system [14]. It has succeeded in solving some autonomous and non-autonomous systems such as a spin particle in a rotating magnetic field with SU(2) algebraic structure (here a $spin - \frac{1}{2}$ particle is viewed as a qubit) [15]. In the present work, using the method of dynamic algebra, we study the properties of the homotrinuclear linear spin cluster with orbital degeneracy (each particle with $s = 1/2$ and $\tau = 1/2$) driven by an external magnetic field. The Hamiltonian is rotationally symmetric in both $\hat{s}$ and $\hat{\tau}$ subspaces in this homotrinuclear linear spin cluster with orbital degeneracy [7]. When the Hamiltonian of the system undergoes a cyclic evolution, the state of a quantum system will acquire a Berry phase of purely geometric origin [16]. The geometric phase was first introduced in the study of the interference of light states of polarization [17]. Since then, there have been numerous investigations on the geometric phase, such as the geometric phase for non-adiabatic, cyclic, non-cyclic and non-unitary evolution [18, 19]. In this paper, the geo-

metric phase and other quantum information of the homotrinuclear linear spin cluster with orbital degeneracy driven by an external magnetic field are calculated exactly based on dynamic algebra. If we input some quantum information into the higher symmetric system which is formed by the unit of the homotriunclear linear spin cluster with twofold orbital degeneracy, the output information will change due to the geometric phase produced by the magnetic field.

The Hamiltonian of the homotrinuclear spin cluster with twofold degenerate orbits ($\tau = 1/2$) and rotational invariance in both $\hat{s}$ and $\hat{\tau}$ space in a magnetic field can be obtained as [7]

$$\begin{aligned}\hat{H} = &\sum_{i=1}^{2}(2\hat{s}_i\cdot\hat{s}_j+1/2)(2\hat{\tau}_i\cdot\hat{\tau}_j+1/2)\\ &+\sum_{i}^{3}B(t)\cdot(\gamma_s\hat{s}_i+\gamma_\tau\hat{\tau}_i)\end{aligned} \tag{1}$$

with

$$\hat{H}_{\rm int} = \sum_{i=1}^{2}(2\hat{s}_i\cdot\hat{s}_j+1/2)(2\hat{\tau}_i\cdot\hat{\tau}_j+1/2), \tag{2}$$

$$\hat{H}_{\rm zeeman} = \sum_{i}^{3}B(t)\cdot(\gamma_s\hat{s}_i+\gamma_\tau\hat{\tau}_i), \tag{3}$$

where the Hamiltonian $\hat{H}_{\rm int}$ is the correlation interaction of adjacent particles, $\hat{H}_{\rm zeemam}$ is the Zeeman interaction of a single centre particle, and $B(t)$ is the rotating magnetic field.

2. Eigen Solutions of $\hat{H}_{\rm int}$

Let $\hat{L}_\alpha = \sum_i \hat{\tau}_i^\alpha$ and $\hat{S}_\alpha = \sum_i \hat{s}_i^\alpha$, where $\alpha = (x,y,z)$ represents the component of the angular momentum $\hat{\tau}$ and spin $\hat{s}$, respectively. Using the commutation rules $\left[\hat{\tau}_i^\alpha,\hat{\tau}_j^\beta\right] = {\rm i}\delta_{ij}\varepsilon_{\alpha\beta\gamma}\hat{\tau}_i^\gamma$ and $\left[\hat{s}_i^\alpha,\hat{s}_j^\beta\right] = {\rm i}\delta_{ij}\varepsilon_{\alpha\beta\gamma}\hat{s}_i^\gamma$ (here $i,j=1,2,3$; if $i=j$, then $\delta_{ij}=1$, otherwise $\delta_{ij}=0$; $\varepsilon_{\alpha\beta\gamma}$ denotes three order anti-symmetric tensors), it is easy to prove

$$\begin{aligned}&\left[\hat{H}_{\rm int},\hat{L}_\alpha\right]=0, \quad \left[\hat{H}_{\rm int},\hat{S}_\alpha\right]=0,\\ &\left[\hat{L}_\alpha,\hat{L}_\beta\right]={\rm i}\varepsilon_{\alpha\beta\gamma}\hat{L}_\gamma, \quad \left[\hat{S}_\alpha,\hat{S}_\beta\right]={\rm i}\varepsilon_{\alpha\beta\gamma}\hat{S}_\gamma.\end{aligned} \tag{4}$$

As (1) has SU(4) or SU(2)⊗SU(2) symmetry [12], the reducible basis of Hamiltonian $\hat{H}$ denotes $\prod_i|s_z,\tau_z\rangle_i = \prod_i|s_z\rangle_i\prod_i|\tau_z\rangle_i$. Let

$$\varphi_0^s(i)=|s_z=1/2\rangle_i,\ \varphi_1^s(i)=|s_z=-1/2\rangle_i,\ \phi^s=\prod_i|s_z\rangle_i,$$

we can get the basis of the $\hat{s}$ space:

$$\begin{aligned}&\phi_1^s=\varphi_0^s(1)\varphi_1^s(2)\varphi_1^s(3), \quad \phi_2^s=\varphi_0^s(1)\varphi_0^s(2)\varphi_1^s(3),\\ &\phi_3^s=\varphi_0^s(1)\varphi_1^s(2)\varphi_0^s(3), \quad \phi_4^s=\varphi_0^s(1)\varphi_0^s(2)\varphi_0^s(3),\\ &\phi_5^s=\varphi_1^s(1)\varphi_1^s(2)\varphi_1^s(3), \quad \phi_6^s=\varphi_1^s(1)\varphi_1^s(2)\varphi_0^s(3),\\ &\phi_7^s=\varphi_1^s(1)\varphi_0^s(2)\varphi_1^s(3), \quad \phi_8^s=\varphi_1^s(1)\varphi_0^s(2)\varphi_0^s(3).\end{aligned}$$

Similarly, let

$$\varphi_0^\tau(i)=|\tau_z=1/2\rangle_i,\ \varphi_1^\tau(i)=|\tau_z=-1/2\rangle_i,\ \phi^\tau=\prod_i|\tau_z\rangle_i,$$

we get the basis of the twofold orbital space:

$$\begin{aligned}&\phi_1^\tau=\varphi_0^\tau(1)\varphi_1^\tau(2)\varphi_1^\tau(3), \quad \phi_2^\tau=\varphi_0^\tau(1)\varphi_0^\tau(2)\varphi_1^\tau(3),\\ &\phi_3^\tau=\varphi_0^\tau(1)\varphi_1^\tau(2)\varphi_0^\tau(3), \quad \phi_4^\tau=\varphi_0^\tau(1)\varphi_0^\tau(2)\varphi_0^\tau(3),\\ &\phi_5^\tau=\varphi_1^\tau(1)\varphi_1^\tau(2)\varphi_1^\tau(3), \quad \phi_6^\tau=\varphi_1^\tau(1)\varphi_1^\tau(2)\varphi_0^\tau(3),\\ &\phi_7^\tau=\varphi_1^\tau(1)\varphi_0^\tau(2)\varphi_1^\tau(3), \quad \phi_8^\tau=\varphi_1^\tau(1)\varphi_0^\tau(2)\varphi_0^\tau(3).\end{aligned}$$

Thus all the 64 bases of the SU(2)⊗SU(2) system can be written as

$$\phi_m=\phi_a^s\phi_b^\tau, \quad a,b\in(1,2,3,4,5,6,7,8). \tag{5}$$

In this Hilbert space, the matrix element of $\hat{H}_{\rm int}$ is $\langle\phi_m|\hat{H}_{\rm int}|\phi_n\rangle$. It is easy to diagonalize $\hat{H}_{\rm int}$ and get the energy levels of $\hat{H}_{\rm int}$:

$E_{\rm int}$:	0	1	2,
degeneracy :	36	24	4.

3. Eigen Solutions of $\hat{H}$ and Geometric Phase

The time-dependent Schrödinger equation ($\hbar=1$) is

$${\rm i}\frac{\partial}{\partial t}|\Psi(t)\rangle = \hat{H}(t)|\Psi(t)\rangle. \tag{6}$$

Using the identity

$$\begin{aligned}&{\rm e}^{-{\rm i}v_\alpha\hat{S}_\alpha}\hat{S}_\beta{\rm e}^{{\rm i}v_\alpha\hat{S}_\alpha} = \hat{S}_\beta\cos v_\alpha+\varepsilon_{\alpha\beta\gamma}\hat{S}_\gamma\hat{S}_\gamma\sin v_\alpha,\\ &{\rm e}^{-{\rm i}v_\alpha\hat{L}_\alpha}\hat{L}_\beta{\rm e}^{{\rm i}v_\alpha\hat{L}_\alpha} = \hat{L}_\beta\cos v_\alpha+\varepsilon_{\alpha\beta\gamma}\hat{L}_\gamma\hat{L}_\gamma\sin v_\alpha,\end{aligned} \tag{7}$$

we can perform a gauge transformation by

$$U_{\rm g}(t) = {\rm e}^{{\rm i}[v_z(t)\hat{S}_z+u_z(t)\hat{\tau}_z]}{\rm e}^{{\rm i}[v_y(t)\hat{S}_y+u_y(t)\hat{\tau}_y]}, \tag{8}$$

where $S_z=\sum_{i=1}^{3}s_i^z$, $\tau_z=\sum_{i=1}^{3}\tau_i^z$. The Schrödinger equation (6) changes into

$${\rm i}\frac{\partial}{\partial t}|\bar{\Psi}(t)\rangle = \hat{\bar{H}}(t)|\bar{\Psi}(t)\rangle, \tag{9}$$

where

$$\begin{aligned}\hat{\bar{H}}(t) &= U_{\rm g}(t)^{-1}\hat{H}U_{\rm g}(t) - {\rm i}U_{\rm g}(t)^{-1}\frac{\partial}{\partial t}U_{\rm g}(t)\\ &= \hat{H}_{\rm int} + f_s(t)\hat{S}_z + f_\tau(t)\hat{L}_z \\ &= \hat{H}_{\rm int} + \hat{\bar{H}}_{\rm zeemam},\end{aligned} \tag{10}$$

$$|\bar{\Psi}(t)\rangle = U_{\rm g}(t)^{-1}|\Psi(t)\rangle, \tag{11}$$

and

$$\begin{aligned}f_s(t) &= \frac{\gamma_s B_y(t)\sin v_y(t) - \gamma_s B_x(t)\cos v_z(t)}{\sin v_y(t)},\\ f_\tau(t) &= \frac{\gamma_\tau B_y(t)\sin u_y(t) - \gamma_\tau B_x(t)\cos u_z(t)}{\sin u_y(t)},\\ \hat{\bar{H}}_{\rm zeeman} &= f_s(t)\hat{S}_z + f_\tau(t)\hat{L}_z.\end{aligned}$$

The parameters satisfy

$$\begin{aligned}&\dot{v}_z\sin v_y + \gamma_s B_x\cos v_y\cos v_z\\ &-\gamma_s B_y\cos v_y\sin v_z + \gamma_s B_z\sin v_y = 0,\end{aligned} \tag{12}$$

$$\dot{v}_y + \gamma_s B_x\sin v_z + \gamma_s B_y\cos v_z = 0, \tag{13}$$

$$\begin{aligned}&\dot{u}_z\sin u_y + \gamma_\tau B_x\cos u_y\cos u_z\\ &-\gamma_\tau B_y\cos u_y\sin\dot{u}_z + \gamma_\tau B_z\sin s u_y = 0,\end{aligned} \tag{14}$$

$$\dot{u}_y + \gamma_\tau B_x\sin u_z + \gamma_\tau B_y\cos u_z = 0. \tag{15}$$

Using (10) and (12)–(15), we obtain the eigenenergy levels and the eigenwave functions of $\hat{\bar{H}}(t)$:

$$\hat{\bar{H}}(t)|\phi_m\rangle = E_m|\phi_m\rangle.$$

The Schrödinger equation (9) has the solution

$$|\bar{\Psi}_m(t)\rangle = {\rm e}^{-{\rm i}\Theta_m(t)}|\phi_m\rangle. \tag{16}$$

It is easy to compute Θ_m:

$$\Theta_m = \int_0^t E_m{\rm d}t' = E^m_{\rm int}t + \int_0^t E^m_{\rm zeeman}{\rm d}t',$$

where

$$\begin{aligned}E^m_{\rm int} &= \langle\bar{\Psi}_m(t)|\hat{H}_{\rm int}|\bar{\Psi}_m(t)\rangle,\\ E^m_{\rm zeeman} &= \langle\bar{\Psi}_m(t)|\hat{\bar{H}}_{\rm zeeman}|\bar{\Psi}_m(t)\rangle.\end{aligned}$$

The orthogonal dynamic non-adiabatic eigenwave functions can be obtained as

$$\begin{aligned}|\Psi_m(t)\rangle &= U_{\rm g}|\bar{\Psi}_m(t)\rangle\\ &= {\rm e}^{-{\rm i}\Theta_m(t)}\\ &\quad\cdot\sum_{a'b'} d^j_{a'a}(-v_y)d^j_{b'b}(-u_y){\rm e}^{{\rm i}(Mv_z+Lu_z)}|\phi_n\rangle,\end{aligned} \tag{17}$$

where $d^j_{a'a}(-v_y)$ and $d^j_{b'b}(-u_y)$ are the Wigner d-functions. M and L denote

$$\begin{aligned}\hat{S}_z|\bar{\Psi}_m(t)\rangle &= M|\bar{\Psi}_m(t)\rangle,\\ \hat{L}_z|\bar{\Psi}_m(t)\rangle &= L|\bar{\Psi}_m(t)\rangle.\end{aligned}$$

The non-adiabatic energy levels are defined as

$$\begin{aligned}E_m(t) &= \langle\Psi_m(t)|\hat{H}|\Psi_m(t)\rangle\\ &= \langle\hat{\bar{\Psi}}(t)|\hat{H}_{\rm int}|\hat{\bar{\Psi}}(t)\rangle + f_s(t)M + f_\tau(t)L\\ &\quad - M\dot{v}_z\cos v_y - L\dot{u}_z\cos u_y,\end{aligned} \tag{18}$$

where

$$\langle\hat{\bar{\Psi}}(t)|\hat{H}_{\rm int}|\hat{\bar{\Psi}}(t) = E_{\rm int}.$$

The above basis (16) satisfies the original time-dependent Schrödinger equation (6), and the general solution of the time-dependent Schrödinger equation can be written as the expansion of the non-adiabatic basis:

$$\begin{aligned}&|\Psi(t)\rangle = \sum_m C_m|\Psi_m(t)\rangle = \sum_m C_m U_{\rm g}|\bar{\Psi}_m(t)\rangle =\\ &{\rm e}^{-{\rm i}\Theta_m(t)}\sum_m C_m\sum_{a'b'} d^j_{a'a}(-v_y)\\ &\qquad\cdot d^j_{b'b}(-u_y){\rm e}^{{\rm i}(Mv_z+Lu_z)}|\phi_n\rangle,\end{aligned} \tag{19}$$

where C_m are the time-dependent expansion coefficients completly determined by the initial condition. The dynamic information of the system is contained in the dynamic adiabatic basis $|\Psi_m(t)\rangle$. The initial condition ($t=0$) is $|\Psi(0)\rangle = U_{\rm g}(0)|\bar{\Psi}_m(0)\rangle$. After time evolution of the wave function from $|\Psi(0)\rangle$ to $|\Psi(t)\rangle$, the total phase or Pancharatnam phase $\Xi^m_{\rm t}(t)$ [17] can be obtained from the Appendix:

$$\begin{aligned}\Xi^m_{\rm t}(t) &= \arg[\langle\Psi(0)|\Psi_m(t)\rangle]\\ &= -\Theta_m(t) + \arctan\left[\frac{A_1B_2 + A_2B_1}{A_1A_2 + B_1B_2}\right],\end{aligned} \tag{20}$$

which comprise of the dynamic component (dynamic phase)

$$\begin{aligned}\Xi^m_{\rm d}(t) &= \int_0^t E_m(t'){\rm d}t'\\ &= \Theta_m(t) - M\int_0^t \dot{v}_z\cos v_y(t'){\rm d}t'\\ &\quad + L\int_0^t \dot{u}_z\cos u_y(t'){\rm d}t'\end{aligned} \tag{21}$$

and the geometric component (geometric phase)

$$\begin{aligned}\Xi_{\mathrm{g}}^{m} &= \Xi_{t}^{m} + \Xi_{d}^{m} \\ &= \arctan\left[\frac{A_1B_2 + A_2B_1}{A_1A_2 + B_1B_2}\right] \\ &\quad - M\int_0^t \dot{v}_z \cos v_y(t')\mathrm{d}t' \\ &\quad + L\int_0^t \dot{u}_z \cos u_y(t')\mathrm{d}t'.\end{aligned} \tag{22}$$

It is clear from the above results that the physical quantities Pancharatnam phase, geometric component, energy levels are dependent on the parameters $v_a(t)$ and $u_a(t)$ of the subspaces [spin and τ subspaces, each of which has SU(2) symmetry] and $\alpha \in (y,z)$. As the parameters are controlled by the time-dependent magnetic field and the magnetic momentums γ_s and γ_τ, we can easily and exactly calculate $v_a(t)$ and $u_a(t)$, if $B(t)$ is given.

4. Solutions of $\hat{H}$ and Phases under a Rotating Magnetic Field

We have shown that the SU(2)⊗SU(2) system in a magnetic field can be calculated by algebraic dynamics. In the following, we consider a counter-clockwise rotating magnetic field, i. e.

$$B(t) = (B\sin\theta\cos\omega t, B\sin\theta\sin\omega t, B\cos\theta), \tag{23}$$

where θ is the angle between the magnetic field and z-axis, and ω is the angular velocity of the rotating magnetic field. Inserting (23) into (12)–(15), we get

$$\dot{v}_z = \dot{u}_z = -\omega t, \tag{24}$$

$$\cos v_y = \frac{\cos\theta - \omega/(\gamma_s B)}{\{1 - 2[\omega\cos\theta/(\gamma_s B)] + [\omega/(\gamma_s B)]^2\}^{\frac{1}{2}}}, \tag{25}$$

$$\cos u_y = \frac{\cos\theta - \omega/(\gamma_\tau B)}{\{1 - 2[\omega\cos\theta/(\gamma_\tau B)] + [\omega/(\gamma_\tau B)]^2\}^{\frac{1}{2}}}. \tag{26}$$

We obtain the non-adiabatic energy levels in the rotating magnetic field from (10)–(18) and (24)–(26):

$$\begin{aligned}E_m(t) &= \langle\Psi_m(t)|\hat{H}|\Psi_m(t)\rangle = E_{\mathrm{int}} \\ &+ \{[1 - 2(\omega/\gamma_s B) + (\omega/\gamma_s B)^2]^{\frac{1}{2}} - \omega\cos v_y(t)\}M \\ &- \{[1 - 2(\omega/\gamma_\tau B) + (\omega/\gamma_\tau B)^2]^{\frac{1}{2}} - \omega\cos u_y(t)\}L,\end{aligned} \tag{27}$$

and the orthogonal dynamic non-adiabatic basis in the rotating magnetic field:

$$\begin{aligned}&|\Psi_m(t)\rangle = \exp(-\mathrm{i}\Theta_m(t)) \\ &\cdot \sum_{a'b'} d^j_{a'a}(-v_y) d^j_{b'b}(-u_y) \exp(-\mathrm{i}(\omega M + \omega L))|\phi^s_{a'}\rangle|\phi^\tau_{b'}\rangle.\end{aligned} \tag{28}$$

Observing (28), after the system in the rotating magnetic field has undergone a cyclic evolution from the initial time ($t = 0$) to the final time ($T = 2\pi/\omega$), each orthogonal dynamic non-adiabatic basis in the rotating magnetic field will acquire a total phase

$$\Xi_{\mathrm{t}}^{m} = -\Theta_m(T) - 2(M + L)\pi. \tag{29}$$

From (21) and (24)–(26) we get the dynamical phase

$$\Xi_{\mathrm{d}}^{m} = \Theta_m(T) + 2\pi(M\cos v_y + L\cos u_y). \tag{30}$$

Obviously, the Berry phase is

$$\Xi_{\mathrm{g}}^{m} = 2\pi(M\cos v_y + L\cos u_y) - 2(M + L)\pi. \tag{31}$$

As $M \neq 0$, the Berry phase of the homotrinuclear spin cluster system with twofold degenerate orbitals can also be written as

$$\Xi_{\mathrm{g}}^{m} = 2\pi M\left[\left(\cos v_y + \frac{L}{M}\cos u_y\right) - \left(1 + \frac{L}{M}\right)\right]. \tag{32}$$

Equations (24)–(26) and (32) uncover that the Berry phase is changed by changing the parameters $\cos v_y$ and $\cos u_y$ which are controlled by the parameters $\omega/(\gamma_\tau B)$, $\omega/(\gamma_s B)$ and θ. The Berry phase changed independently in the subspaces of spin and τ, each of which has the similar form of the Berry phase in the pure spin space [15]. The difference of the coefficient γ_τ and γ_s will induce the variance or changing speed of the Berry phases in the subspaces. Let $N = \gamma_s/\gamma_\tau$ and $\Omega = \gamma_s B$, thus (26) can be written as

$$\cos u_y = \frac{\cos\theta - N\omega/\Omega}{[1 - 2(N\omega\cos\theta/\Omega) + (N\omega/\Omega)^2]^{\frac{1}{2}}}.$$

It is obvious from Figs. 1 and 2 that, in addition to ω/Ω, the parameter N (here $N = \gamma_s/\gamma_\tau$) also has an important influence on the changing speed of the Berry phase. The Berry phase decreases, when N increases. We can change the parameter ω/Ω to obtain the different Berry phase. In contrast N is a constant that only

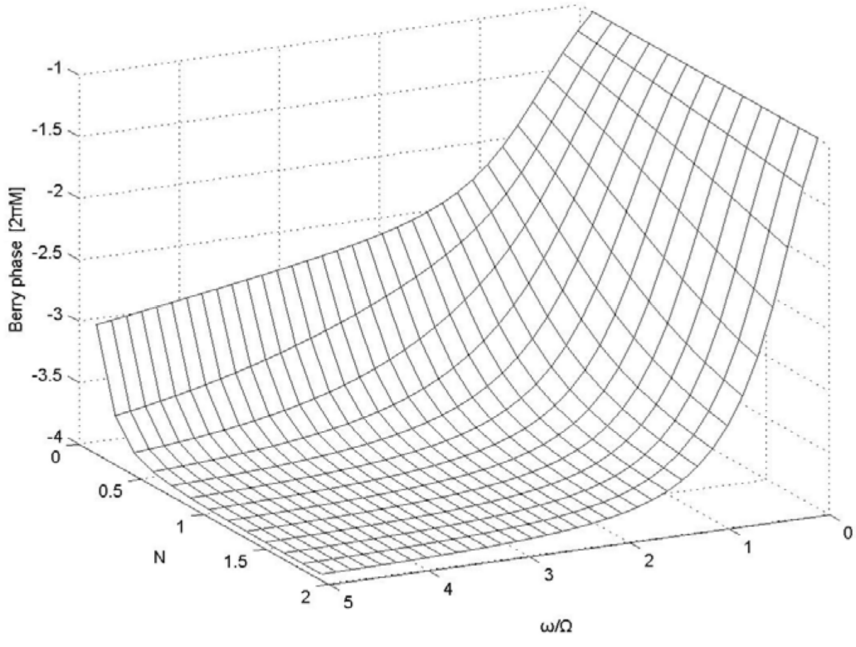

Fig. 1. Change of the Berry phase for $\theta = \pi/3$, $L/M = 1$, $\omega/\Omega \in (0,5)$, $N \in (0,1,2)$; the magnetic field rotates around the z-axis.

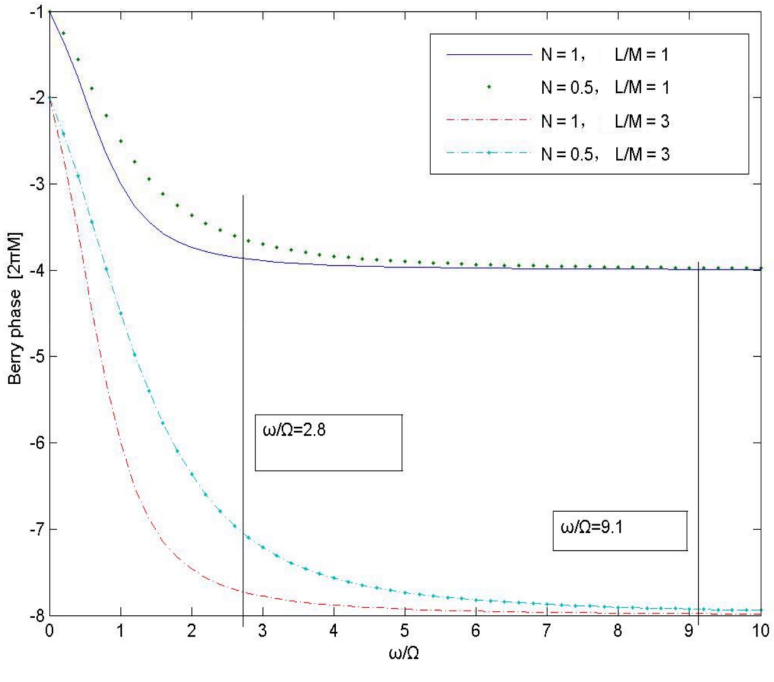

Fig. 2. Change of the Berry phase at special coefficients of γ_s/γ_τ and L/M from the exact expressions equation (24).

depends on the property of the material. From Fig. 2 we can find that in a small range of ω/Ω ($|\omega/\Omega| \leq 2.7$) the Berry phase decreases rapidly as ω/Ω increases, whereas in the range $|\omega/\Omega| \geq 2.7$ the changing speed of the Berry phase slows down. The curves also show that the Berry phase of the same coefficient is almost the same in the large range of ω/Ω ($|\omega/\Omega| \geq 9.1$), that is to say the parameter N has nearly no influence on the Berry phase in the large range of ω/Ω ($|\omega/\Omega| \geq 9.1$).

5. Conclusion

By using the method of dynamic algebra, we have obtained the solutions of the homotrinuclear spin cluster with twofold degenerate orbitals in a time-dependent magnetic field. Based on the theory of SU(2) dynamic algebra, the non-adiabatic energy levels and phases have been calculated. From the solutions, we have found that in a rotating magnetic field the change of the Berry phase is influenced by the parameter $N = \gamma_s/\gamma_\tau$ (derived from the Berry phase, changing independently in the subspaces of spin and τ) in addition to ω/Ω. In the rotating magnetic field, the Berry phase changes rapidly in a small range of ω/Ω ($|\omega/\Omega| \leq 2.7$) and the Berry phase decreases rapidly when N increases, whereas in the large range of ω/Ω ($|\omega/\Omega| \geq 9.1$) the parameter N has nearly no influence on the Berry phase. These results may be helpful in the research of quantum calculation and the design of quantum computer. This work has shown that the method of algebraic dynamics is a useful tool for autonomous and non-autonomous systems with dynamic symmetry.

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Appendix

The coefficients in (20) and (22) are

$$\begin{aligned}
A_1 &= \cos\frac{v_y(t)+v_y(0)}{2}\left\{\cos^2\frac{v_y(t)-v_y(0)}{2}[\cos(m_1v_z(t))\cos(m_2v_z(t))\sin(m_3v_z(t))\right.\\
&\quad +\cos(m_3v_z(t))\cos(m_1v_z(t))\sin(m_2v_z(t))+\cos(m_2v_z(t))\cos(m_3v_z(t))\sin(m_1v_z(t))]\\
&\quad \left.-\cos^2\frac{v_y(t)+v_y(0)}{2}\sin(m_1v_z(t))\sin(m_2v_z(t))\sin(m_3v_z(t))\right\},\\
A_2 &= \cos\frac{u_y(t)+u_y(0)}{2}\left\{\cos^2\frac{u_y(t)-u_y(0)}{2}[\cos(l_1u_z(t))\cos(l_2u_z(t))\sin(l_3u_z(t))\right.\\
&\quad +\cos(l_3u_z(t))\cos(l_1u_z(t))\sin(l_2u_z(t))+\cos(l_2u_z(t))\cos(l_3u_z(t))\sin(l_1u_z(t))]\\
&\quad \left.-\cos^2\frac{u_y(t)+u_y(0)}{2}\sin(l_1u_z(t))\sin(l_2u_z(t))\sin(l_3u_z(t))\right\},\\
B_1 &= \cos\frac{v_y(t)-v_y(0)}{2}\left\{\cos^2\frac{v_y(t)+v_y(0)}{2}[\cos(m_1v_z(t))\sin(m_2v_z(t))\sin(m_3v_z(t))\right.\\
&\quad +\cos(m_3v_z(t))\sin(m_1v_z(t))\sin(m_2v_z(t))+\cos(m_2v_z(t))\sin(m_3v_z(t))\sin(m_1v_z(t))]\\
&\quad \left.-\cos^2\frac{v_y(t)-v_y(0)}{2}\cos(m_1v_z(t))\cos(m_2v_z(t))\cos(m_3v_z(t))\right\},\\
B_2 &= \cos\frac{u_y(t)-u_y(0)}{2}\left\{\cos^2\frac{u_y(t)+u_y(0)}{2}[\cos(l_1u_z(t))\sin(l_2u_z(t))\sin\tau(l_3u_z(t))\right.\\
&\quad +\cos(l_3u_z(t))\sin(l_1u_z(t))\sin(l_2u_z(t))+\cos(l_2u_z(t))\sin(l_3u_z(t))\sin(l_1u_z(t))]\\
&\quad \left.-\cos^2\frac{u_y(t)-u_y(0)}{2}\cos(l_1u_z(t))\cos(l_2u_z(t))\cos(l_3u_z(t))\right\}.
\end{aligned}$$

Here m_i ($i = 1,2,3$) is the z-component of the spin momentum of the i-th particle and l_i ($i = 1,2,3$) is the z-component of the orbital momentum of the i-th particle.

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